Introduction to Magnetic Characterization of Solids

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Agenda

- Basic definitions and principles
- Magnetic classification of materials
- Daiamagnetism
- Paramagnetism
- o Ferromagnetism

Basic Principles

 Magnetism originates from quantum behaviour of elementary particles, from microscopic current loops and magnetic dipoles. Dipoles, unlike their analogues in electrostatics, cannot be separated into + and – magnetic charges (monopoles). There is also no analogue of the free conduction electron in magnetism. But in many cases magnetically polarised or polarisable materials are close analogues of electrically polarisable materials (dielectrics). Finally, nor current loops, neither dipoles can explain ferromagnetism, which arises from a non-classical, quantum-mechanic phenomenon – electron spin.



Magnetic field is described by its intensity *H* or induction *B*. These parameters are defined not by their cause, but by their effect, i.e. the forces they exert on physical objects. Magnetic field acts by torque moment on a compass needle (macroscopic dipole) tending to align its axis parallel with *H* or *B* – magnetostatic definition. Moreover, magnetic field exerts a Lorentz force on a moving charged particle (or currant *I*), the force being at right angles to both the field direction and the current flow – electrodynamic definition. However, these definitions do not explain the source of *H* or *B*. This is given by Coulomb's law or the Biot-Savart law.



• **B** is augmented field which includes **H** and also macroscopic magnetization **M**.

 $\boldsymbol{B} = \mu_0 (\boldsymbol{H} + \boldsymbol{M})$ in SI, or $\boldsymbol{B} = \boldsymbol{H} + 4\pi \boldsymbol{M}$ in cgs. and

$$\boldsymbol{M} = \mu_0 \kappa \boldsymbol{H}$$

- Microscopic dipole moment μ is associated with individual atoms. M dipole moment per unit volume.
- **B** is sometimes considered as more fundamental. It depends on the rate of change of magnetic flux (often called flux density, lines of force cutting a particular area). Thus, sum of **M** and **H** is important namely inside ferromagnetics. Outside a magnetic material M = 0, **B** and **H** are parallel and $B/H = \mu_0$, the permeability of vacuum (= $4\pi \times 10-7$ H/m; = 1 in cgs). Numerical values of *B* (in Tesla in SI and gauss in cgs) and *H* (in A/m in SI and Oersted in cgs) are very different for the same field.

Magnetic moment of dipole is $\mu = Q_m dk$. At a point P the magnetic field is given by <u>Coulomb law</u> as:

$$\boldsymbol{H} = \frac{\boldsymbol{B}}{\mu_0} = \frac{1}{\mu_0} \frac{\mu_0}{4\pi} \left(\frac{Q_m}{r_1^2} \hat{r}_1 - \frac{Q_m}{r_2^2} \hat{r}_2 \right) = \frac{\mu}{4\pi r} \left(2\cos\theta \ \hat{r} + \sin\theta \,\hat{\theta} \right)$$

and along the axis of **µ** by:

$$\boldsymbol{H}_{axial} = \frac{2\mu}{4\pi \ z^3} \hat{k}$$

Magnetic Moment of a Dipole



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Magnetic Moment of a Loop



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Magnetic Moment of a Loop

In the case of circular current loop, *H* is determined by <u>Biot-Savart</u> law (point P is on the axis of the loop):

$$\boldsymbol{H}_{axial} = \oint \frac{I \, dl \times \hat{r}}{4\pi \ r^2} = \oint d\boldsymbol{H} = \frac{I \cos \psi}{4 \ \pi \ (a^2 + z^2)} \oint dl \ \hat{k} = \frac{2I\pi \ a^2}{4\pi \ (a^2 + z^2)^{3/2}} \hat{k}$$

which at large distances reduces to:

$$\boldsymbol{H}_{axial} = \frac{2I\pi \ a^2}{4\pi \ z^3} \hat{k} = \frac{2\mu}{4\pi \ z^3} \hat{n}$$

The two equations for H_{axial} are equivalent, provided dipole moment μ is replaced by $I\pi a^2 n$.

Bohr Model of an Atom

An "s" electron has a circular orbit and its orbital magnetic moment is defined as: $IA\mathbf{n} = (e\omega/2\pi)\pi r^2\mathbf{n}$ A is the loop area.

This is always parallel to orbital mechanical momentum vector:

 $\boldsymbol{L} = \boldsymbol{m}_{e}r^{2}\omega\boldsymbol{n}$ In a magnetic field \boldsymbol{H} , the torque $\mu_{0}\boldsymbol{\mu} \times \boldsymbol{H}$ causes \boldsymbol{L} and $\boldsymbol{\mu}$ to precess like a gyroscope around \boldsymbol{H} .

Basic Principles of Quantum Mechanics

- Energetic states of electron are given by solution of Schrödinger wave equation, i.e. by wave functions, which describe (their square) the probability of finding the electron at certain position (in spherical coordinates). These wave functions involve quantum numbers (<u>n, I, m,</u> <u>s</u>), i.e. energy cannot change continuously, but in discrete steps – quanta.
- Pauli's principle: each electron has a different set of quantum numbers, representing discrete value of energy, angular momentum, magnetic moment and spin:
- o n specifies the orbital shell, n = 1, 2, ...
- *I* specifies the orbital angular momentum **L** (*I* = 0 for s orbit, *I* = 1 for p orbit, etc., $0 \le I \le (n-1)$; $L = \hbar \sqrt{I(I + 1)}$
- m specifies the component of **L** in the direction of applied magnetic field $(-l \le m \le +l)$; $LH = ml\hbar$; MIH = ml mB
- s specifies the spin angular momentum **S** in the direction of **H** ($s = \pm \frac{1}{2}$).

Basic Principles of Quantum Mechanics

The elementary unit (quantum) of magnetic moment is **Bohr magneton**

 $\mu_{\rm B}$ = eħ/2 $m_{\rm e}$ = 9.274 × 10⁻²⁴Am² or 9.274 × 10⁻²¹ emu and in case of single electron on circular orbit

 $\mu_{\rm I} = m \ \mu_{\rm B}$ and $\mu_{\rm s} = \mu_{\rm B}$

Atoms of most elements have only one unpaired electron or no unpaired electrons. Their permanent magnetic moments are small or non-existent. Atoms of transition metals (Cr, Mn, Fe, Ni, Co) have unpaired 3d electrons (n = 3, l = 2) and thus larger moments. They all have filled 4s valence shell but only partly filled 3d shells.

Fe – 26 electrons, $1s^22s^22p^63s^23p^63d^64s^2$

3d – partly filled, following the **Hund's principle**, total $S = (5 \times \frac{1}{2}) - \frac{1}{2}$

 $L = \text{sum of } I_i \text{ is maximised } (2+1+0-1-2)+2 = 2$

Hund's (Building) and Pauli's (Exclusion) Principles



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Hund's (Building) Principle in Iron



Diamagnetism

- Moving charge electron experiences in a magnetic field *B* Lorentz force e*v* × *B*. As a result, due to gyroscopic effect the orbit axis makes precession about *B* Larmor precession, which represents additional motion of the electric charge, producing a negative induced magnetic moment.
- Magnetic susceptibility:

 $\chi_d = M/H = (n\mu_d)/(B/\mu_0) = -(\mu_0 n Z e^2 r^2)/(6m_e)$

- *n* is the number of atoms, *Z* number of electrons, and *r* is the average orbit radius. This is classical Langevin definition of diamagnetism.
- Most non-iron bearing minerals, such as quartz, calcite, feldspars, as well as water, are purely diamagnetic.
- Small, negative and temperature independent
- This property is proper of all materials.

Daiamgnetism



Daiamgnetism



Paramagnetism

At room temperature spontaneously disordered permanent magnetic moments (of atoms) are (partially) aligned in the direction of applied field **B**. Only thermal perturbations prevent perfect alignment of the moments with **B**.

In classical theory, magnetic moment \boldsymbol{M} of an ensemble of atomic moments $\boldsymbol{\mu}$ is determined by

 $\boldsymbol{M} = n\mu L(\alpha) = n\mu \left[\operatorname{coth}(\alpha) - (1/\alpha) \right]$

where $\alpha = \mu B/kT$ and $L(\alpha)$ is the so called Langevin function. The basic meaning is that **M** is positive, i.e. **M** and **B** are parallel, and decreases with increasing temperature.

Because atomic moments μ are small, for most fields and temperatures $\mu B << kT$, the alignment is slight and **M** is a fracture of theoretical total saturation moment $n\mu$. At ordinary temperatures, a field of some 100 T is needed to reach this saturation.

Paramagnetism

Magnetic susceptibility: By expanding $L(\alpha)$ into series and neglecting "very" small members, we get paramagnetic susceptibility as

$$\chi_{\rm p} = M/H = (n\mu^2\mu_0)/(3kT) = C/T$$

Curie law, $C = (n\mu^2\mu_0)/(3k)$ is Curie constant.

Most iron-bearing sulphides, carbonates and silicates are paramagnetic, with susceptibilities (absolute values) 10÷100 times larger than those of diamagnetic minerals.

Paramagnetism



Ferromagnetism

Field of some 10^{-2} T (100 G) - enough to reach saturation magnetization in some materials. Moreover, when the field is removed, a non-zero remanent magnetization remains. Weiss in 1907 postulated the existence of an internal "molecular" field $H_m = \lambda M$ so strong that it almost perfectly aligns atomic moments.

If there is no external field, H_m is the only acting field on μ and

$$E_m = -\mu_0 \mu \, \boldsymbol{H}_m = -\mu_0 \mu \, \lambda M \cos \phi \Rightarrow M = n \mu L(\alpha)$$

with $\alpha = (\mu_0 \mu \ \lambda M)/(kT) \Rightarrow M = (kT\alpha)/(\mu_0 \mu \ \lambda)$

With increasing temperature, decrease in M causes also decrease in H_m , thus provoking further decrease in M. At Curie temperature T_c – transition from spontaneously ordered ensemble to paramagnetic disorder.

Simple estimate of H_m gives value of about 8.5×10^6 Oe, which represents such a strong field that it cannot be due to magnetostatic interactions between neighboring atoms (this is about 3000 Oe).

Ferromagnetism

For *T* above T_c , spontaneous alignment of magnetic moments vanishes due to thermal excitations and susceptibility behaves like in case of paramagnetism, but with individual atomic moments coupled rather than individual.

Туре	Arrangement	Lattice	Resultant
true ferromagnetic (Fe, Ni, Co)	alignment within lattice	$ \uparrow \uparrow \uparrow \uparrow $	1
antiferromagnetic	sublattices, A & B, aligned but antiparallel, equal	$\uparrow \downarrow \uparrow \downarrow$	(none)
ferrimagnetic	sublattices, A & B, aligned antiparallel, unequal	$\uparrow \downarrow \uparrow \downarrow$	1
canted antiferromagnetic	sublattices, A & B, canted equal	$\land \land \land \land$	\rightarrow

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Ferromagnetism



Ferromagnetism - Domains



Para - Ferromagnetism



??? - ??? - ???magnetism

